Name: _____ Class: _____ Date: _____ ID: A

WS 3 - 2.3 Continuity

Short Answer

1. (a) Find each point of discontinuity. (b) Which of the discontinuities are removable?

$$f(x) = \begin{cases} \dfrac{1}{x-1}, & x < 1 \\ x^3 - 2x + 5, & x \geq 1 \end{cases}$$

2. Find $\lim\limits_{x \to 1} \cos \pi x$

3. Find a value for a so that the function

$$f(x) = \begin{cases} 4 - x^2, & x < -1 \\ ax^2 - 1, & x \geq -1 \end{cases}$$

4. If $\lim\limits_{x \to c} f(x) = 27$ and $\lim\limits_{x \to c} g(x) = 3$, find the following

   (a) $\lim\limits_{x \to c} \sqrt[3]{f(x)}$

   (b) $\lim\limits_{x \to c} \left(f(x)\right)^{2/3}$

5. Is $f(x) = \sin(x^3 + 1)$ continuous? Why?

6. Let $f(x) = \begin{cases} x^2 - 4x + 5, & x < 2 \\ 4 - x, & x \geq 2 \end{cases}$, find

   (a) $\lim\limits_{x \to 2^-} f(x)$

   (b) $\lim\limits_{x \to 2^+} f(x)$

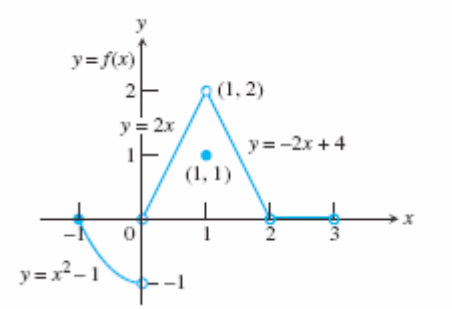
   (c) $\lim\limits_{x \to 2} f(x)$

   (d) $f(2)$

7. Sketch a possible graph for a function f if $f(3)$ exist but $\lim\limits_{x \to 3} f(x)$ does not.

8. Use the function f defined and graphed below to answer the questions.

$$f(x) = \begin{cases} x^2 - 1, & -1 \leq x < 0 \\ 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x < 3 \end{cases}$$



   (a) Does $f(-1)$ exist?

   (b) Does $\lim\limits_{x \to -1^+} f(x)$ exist?

   (c) Does $\lim\limits_{x \to -1^+} f(x) = f(-1)$?

   (d) Is f continuous at $x = -1$

   (e) Is f continuous at $x = 1$? Why?

   (f) Is f defined at $x = 2$?

9. If $\lim_{x \to c} f(x) = 2$ and $\lim_{x \to c} g(x) = 3$, find the following
   (a) $\lim_{x \to c} \left[4f(x) \right]$
   (b) $\lim_{x \to c} \dfrac{f(x)}{g(x)}$

10. Find the limit $\lim_{x \to -\infty} \dfrac{3x^2 - x^3 + 5x + 1}{x^2 - 8}$

11. Let $f(x) = \begin{cases} \dfrac{x+2}{2}, & x \le 3 \\ \dfrac{12 - 2x}{3}, & x > 3 \end{cases}$, find
   (a) $\lim_{x \to 3^+} f(x)$
   (b) $\lim_{x \to 3^-} f(x)$
   (c) $\lim_{x \to 3} f(x)$

12. (a) Find each point of discontinuity. (b) Which of the discontinuities are removable?
   $f(x) = \begin{cases} 3 - x, & x < 2 \\ \dfrac{x}{2} + 1, & x > 2 \end{cases}$

13. Find the limit graphically $\lim_{x \to 0} \left(5 - x^2 \cos \dfrac{1}{x} \right)$

14. Find
   (a) $\lim_{x \to \infty} \dfrac{|4x + 3|}{-2x + 6}$
   (b) $\lim_{x \to -\infty} \dfrac{|4x + 3|}{-2x + 6}$

15. Is there a zero in the interval [0, 1] of the function $f(x) = x^3 + 3x - 2$. Do not use a calculator.

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Answer Section

SHORT ANSWER

1. (a) $x = 1$
   (b) Not removable, it's an infinite discontinuity
2. -1
3. 4
4. (a) 3
   (b) 9
5. Yes
6. (a) 1
   (b) 2
   (c) No limit
   (d) -1
7. -
8. (a) yes
   (b) yes
   (c) yes
   (d) yes
   (e) no
   (f) no
9. (a) 8
   (b) $\frac{2}{3}$
10. ∞
11. (a) $\frac{5}{2}$
    (b) 2
    (c) DNE
12. (a) $x = 2$
    (b) Not removable, the one-sided limits are different
13. 5
14. (a) -2
    (b) 2
15. $f(0) = -2$ amd $f(1) = 2$
    By the IVT a zero exist.